Tunable N-Path Mismatch Shaping for Multibit Bandpass Delta-Sigma Modulators

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Abstract—Many radio applications require the use of programmable bandpass $\Delta \Sigma$ converter. In the digital-to-analog converter (DAC) used within a $\Delta \Sigma$ converter, non-linearities created by DAC element mismatch error can be spectrally shaped to fall outside the signal band. The mismatch shapers within these converters thus also need to be programmable in order to follow the signal band. This paper proposes a new technique that tunes the center frequency of a mismatch noise transfer function while using an arbitrary mismatch shaping algorithm.

I. INTRODUCTION

Modern wireless systems are expected to handle an increasing number of wireless standards. As a result, direct-digital synthesis of the intermediate frequency (IF) signal is becoming more attractive [1] [2] [3]. In particular, bandpass $\Delta \Sigma$ modulators are rapidly becoming the data converter of choice for these applications due to their high linearity over a narrow signal-bandwidth, as shown in Figure 1 [4].

An important feature of software-defined radio is the ability to tune the IF to multiple frequencies. Using a bandpass $\Delta \Sigma$ modulator with a fixed signal-band location would require a very wide signal-bandwidth that encompasses all possible IF settings. However, a bandpass $\Delta \Sigma$ modulator with a programmable noise-transfer-function (NTF) would allow the use of relatively low over-sampling ratios (OSR).

Higher-order modulators with multi-bit quantizers provide a means for achieving very high signal-to-noise ratios. However, mismatch errors in the multi-bit DAC limits the linearity at the output, requiring some form of correction to linearize the DAC transfer function. Bandpass mismatch shapers need to be able to linearize the DAC mismatch wherever the signal band location is programmed.

This paper describes a technique to program the center frequency of the mismatch-shaper over the full tuning range of the bandpass $\Delta \Sigma$-modulator. Section II describes previously reported techniques. Section III describes the proposed tunable $N$-path algorithm. Simulation results are shown in Section IV, and the paper is concluded in Section V.

II. PREVIOUS WORK

In any $\Delta \Sigma$-ADC or $\Delta \Sigma$-DAC, the linearity of the embedded DAC limits the overall linearity of the $\Delta \Sigma$ converter. The inevitable variation in device parameters leads to deviations of individual quantizer levels from their ideal values. Mismatch shaping is one technique for alleviating the effects of mismatch errors [4] [5].

Most mismatch-shaping techniques consist of re-arranging the encoded DAC input such that the noise power in the signal band is reduced while keeping the signal constant intact. Reported techniques include methods such as Dynamic Element Matching (DEM), Clocked Averaging (CLA), Individual Level Averaging (ILA) and Data-Weighted Averaging (DWA) [4].

Although these techniques are relatively simple to implement in hardware, at best they are able to achieve first-order noise shaping. Furthermore, it is difficult to program the locations of the signal bands.

A. Vector-Based Shaping

A technique for achieving higher-order mismatch-shaping has been described in [6] and [7]. These vector-quantization techniques involve the use of individual mismatch-shaping circuits for driving each bit of the DAC input thermometer code. Each mismatch-shaper is an independently operated $\Delta \Sigma$ modulator loop with a bandpass modulator noise-transfer function (NTF), which can be made programmable. This approach suffers from the high complexity of hardware required to implement the modulator array, and concerns about stability with each individual mismatch-shaping loop.

B. Tree-based Selection Structure

Another approach to achieving a higher-order mismatch-shaping response is described in [8]. This structure consists of
a tree of switching blocks that acts as a router for directing each bit of the DAC input thermometer code to each DAC unit-element. Each switching decision is made according to a spectrally shaped number sequence generated by a ∆Σ-modulator. As a result, it is possible to achieve higher orders of noise shaping while arbitrarily centering the signal band in the passband by using a programmable NTF. This approach also suffers from a high implementation complexity and concerns about stability, although it can be more efficient than the vector-based technique.

C. N-Path Multirate Filter

An elegant technique for applying baseband mismatch-shaping algorithms to bandpass modulators is described in [9] and [10]. This involves using the N-path multirate filter property z \rightarrow z^N to shift the response of a given baseband mismatch shaping transfer function to a passband location. Figure 2 shows the basic structure of this approach.

For example, this approach can be applied to a first-order mismatch shaping response (such as DWA), as given by

\[ H(z) = 1 - z^{-1} \]  \hspace{1cm} (1)

Using a 4-path filter z \rightarrow z^4 yields a mismatch response with zeros located at

\[ z = 1, \pm \frac{F_s}{4}, \pm \frac{F_s}{2} \]  \hspace{1cm} (2)

This mismatch shaping response can be used in a bandpass modulator with the signal band centered at \( F_s/4 \).

III. TUNABLE N-PATH MISMATCH SHAPING

The vector quantizer technique (described in Section II-A) and tree-structure technique (described in Section II-B) can each handle a programmable signal-band location. However, both techniques suffer from high hardware complexity and the potential for loop instability. This section describes the proposed technique of applying the N-path filter principle described in Section II-C, such that the signal band location can be tuned.

There have been numerous efforts at applying mismatch-shaping to bandpass ∆Σ modulators which have a fixed location of the signal passband, usually \( F_s/4 \). When the N-path technique is applied, the mismatch transfer function is compressed and replicated \( N \) times around the unit-circle in the \( z \)-domain. The choice of placing the signal-band at \( F_s/4 \) makes it convenient to apply the N-path filter principle with \( N \) chosen as 2 or 4 [9] [10] [11].

A. Tunable Signal Band

This paper proposes a technique whereby the N-path filter principle is applied using variable values of \( N \). Tuning capability is achieved by the fact that each different value of \( N \) will yield a different set of mismatch-shaper locations around the unit circle. With a given value of \( N \), the bandpass ∆Σ modulator can operate with the signal-band centered near any of the \( N \) locations around the unit circle. For example, Figure 3(a) shows the transfer function zeros that result from a 4-path transform \( z \rightarrow z^4 \), and Figure 3(b) shows the result of an 8-path transform \( z \rightarrow z^8 \).

Thus, at a given bandpass ∆Σ modulator tuning frequency setting, the number of paths \( N \) should be chosen such that the final mismatch transfer function sufficiently suppresses the mismatch noise near this tuning frequency.

B. Band Compression

One consequence of using the N-path transform is the factor-of-\( N \) compression the prototype mismatch transfer function undergoes. This causes the usable bandwidth over which effective mismatch shaping occurs is reduced by a factor of \( N \). For example, the magnitude response of the mismatch transfer function with \( N = 4 \) is shown in Figure 4(a), while that of \( N = 8 \) is shown in Figure 4(b).

A tunable bandpass ∆Σ modulator might have signal bands centered at \( F_c \), where

\[ F_c[k] = \frac{k \cdot F_s}{OSR}, k = 0, 1, 2, .., OSR. \]  \hspace{1cm} (3)
The band-center frequencies of the mismatch shaper’s $N$-path image should ideally also be located at exactly $F_c$ (as given by Equation 3). However, in order to minimize signal-band compression in the $N$-path response of the prototype mismatch shaper, the value of $N$ itself should be minimized. Alternatively, the prototype mismatch transfer function stop-band should be maximized.

A simple optimization routine can be employed to obtain the maximum value of $N$ required to achieve an adequately close $N$-path image of the mismatch transfer function for each setting of $F_c$.

C. Prototype Mismatch Shaper

The mismatch-shaper prototype can determine the $N$-path image locations for a given value of $N$. Figure 5 shows the 3-path transform $z \rightarrow z^3$ applied to (a) low-pass mismatch-shaper prototype $H(z) = 1 + z^{-1}$ and to (b) high-pass mismatch-shaper prototype $H(z) = 1 - z^{-1}$.

In the case of a first-order high-pass prototype $H_{hp}(z) = 1 - z^{-1}$, the 3-path zeros are located at

$$z = 1, -\frac{1}{2} + j\frac{\sqrt{3}}{2}.$$  

(4)

However, using a first-order low-pass prototype $H_{lp}(z) = 1 + z^{-1}$, the 3-path zeros are located at

$$z = -1, 1, \frac{1}{2} + j\frac{\sqrt{3}}{2}.$$  

(5)

From Figure 5, it is evident choosing a different prototype mismatch shaper can reduce the number of paths, $N$, required for a given signal band tuning frequency.

D. Implementation Simplicity

The $N$-path filter can be implemented as shown in Figure 2. Alternatively, the baseband mismatch-shaping prototype block can be multiplexed between all paths to run at the oversampled rate, providing each path’s state is saved separately. The $N$-path technique is particularly efficient when a lower-complexity prototype mismatch-shaper such as DWA is used.

Each setting of the signal-band center frequency requires a corresponding value of the number of paths, $N$, and a binary selection bit, $P[k]$ indicating the type of prototype mismatch shaper to be used. These tuning pairs $(N[k], P[k])$ can be stored in a small lookup-table, and indexed by the tuning frequency $F_c[k]$.

The low-pass prototype mismatch shaper ($H(z) = 1 + z^{-1}$) and the high-pass version ($H(z) = 1 - z^{-1}$), can be implemented using simple element-rotation algorithms. Figure 6 shows example sequences for each prototype mismatch shaper.

IV. Simulation Results

A bandpass $\Delta\Sigma$ modulator has been synthesized according to the specifications given in Table I. The prototype mismatch shapers used within the $N$-path structure are both based on element-rotation. Figure 7 shows the number of paths at each of the 64 tuning settings, along with the corresponding location of the first-order mismatch shaper zero.

TABLE I
TUNABLE BANDPASS $\Delta\Sigma$ MODULATOR SPECIFICATIONS

<table>
<thead>
<tr>
<th>Loop Filter Order</th>
<th>OSR</th>
<th>Quantizer Levels</th>
<th>SNR (no mismatch)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>64</td>
<td>9</td>
<td>86 dB</td>
</tr>
</tbody>
</table>

Figure 8 shows the hardware complexity of the first-order tunable $N$-path implementation over the entire tuning range. Also shown in Figure 8 is the hardware complexity of each reported first-order mismatch shaper, which operate at fixed frequency locations.

The tunable $N$-path technique must be implemented using the maximum number of paths over the desired tuning range. The complexity of implementing the full tuning range is shown with the dashed line in Figure 8.

Figure 9 shows performance results from three different levels of mismatch-error: 0.8%, 2% and 3%. The in-band signal-to-noise ratio (SNR) was computed and plotted across 64 values of the band-center frequency $F_c[k]$, corresponding to $OSR = 64$ and Equation 3.

In each case shown in Figure 9, there are three sets of data shown together:

- The dashed-line shows the maximum achievable SNR when there is no DAC mismatch error.
Fig. 7. Lookup table information for tunable $N$-path implementation

Fig. 8. Hardware complexity of tunable $N$-path technique

- The solid line dotted with asterisks shows the SNR performance at each point when the DAC suffers from the indicated mismatch error, but mismatch-shaping is performed.
- The solid line shows the SNR when there is DAC mismatch error, but no mismatch-shaping is performed.

As shown in Figure 9, the tunable $N$-path mismatch-shaping technique yields consistent improvement over the entire range of tuning frequencies $F_c[k]$.

V. CONCLUSION

A new technique for implementing a tunable mismatch shaper has been presented. Simulation results with first-order mismatch shaping have shown the effectiveness of this approach over the entire tuning range. In addition, this technique provides a great deal of implementation simplicity and flexibility, as any fixed-frequency mismatch shaper prototype can be used within the tunable $N$-path structure.

It is possible to improve performance by implementing the tunable $N$-path technique using a higher-order mismatch shaper as the prototype, but this would come at the expense of additional implementation complexity.

Higher-order functions might also require multiple computational steps, adding to the overall cycle latency through the mismatch shaper. Higher latency contributes to instability when the mismatch shaper is used inside the noise-shaping loop of a $\Delta\Sigma$ analog-to-digital converter (ADC).

REFERENCES